Scattering of 19.2 GeV/c Protons on Free Protons in Nuclear Emulsion

V. A. BULL* AND D. A. GARBUTT
Physics Department, University College, London, England
(Received 17 December 1962)

An experimental study has been made of the elastic scattering of protons by free protons at a laboratory momentum of 19.2 GeV/c. The proton beam was directed at perpendicular incidence into water-loaded nuclear emulsions which were used as both target and detector. For elastic p-p scattering the total cross section was found to be 9.4 ± 1.3 mb. At zero angle the measured value of the differential cross section was higher than that predicted by the optical theorem, indicating the existence of a real part of the forward scattering amplitude or a spin dependence of the total proton-proton cross section. An analysis of other experimental data above 2.5 GeV is presented confirming the above result and is compared with the predictions of a Regge-pole model at zero angle.

INTRODUCTION

THE study of elastic proton-proton scattering at small momentum transfers (i.e., $K_c \sin\theta_c < 1\times10^{+13}$ cm^{-1})[1] gives information on the interference between Coulomb and nuclear scattering, the extrapolated zero-angle cross section as compared with the predictions of the optical theorem, and the various optical-model parameters which describe the scattering at high energies.

In particular, at energies between 2.5 and 10 GeV, a number of authors[2–6] have reported experiments in which extrapolated zero-angle cross sections were in excess of that given by the optical theorem. This would require the existence of a real part of the scattering amplitude or a spin-dependent interaction.

It is therefore of interest to extend the measurements to higher energies and the present work is an investigation of small angle proton-proton scattering at 19.2 GeV/c.

EXPERIMENTAL PROCEDURE

Three stacks of four Ilford G5 glass-backed emulsions (10 cm$\times$10 cm$\times$600 μm) were exposed to the 19.2 GeV/c momentum analyzed proton beam ($C4$) scattered out from the CERN Proton Synchrotron. In each stack the beam entered the pellicles, arranged as shown in Fig. 1, perpendicular ($\pm0.1°$) to the emulsion surfaces. Before exposure the central pair of Emulsions B and C had been soaked in demineralized water at 20°C for $4\frac{1}{2}$ hours. Emulsion B was the target, C was used to follow the proton recoils if they left B, and the dry emulsions A and D were used only in the measurement of the scattering angle, θ.

SCANNING

The water-loaded emulsions had a very low grain density ($\sim$8/100 μm) making area scanning for recoil protons from elastic scatters, as described by Lyubimov *et al.*,[3] very inefficient. A line-scanning technique[7] was employed, therefore, to detect small-angle scatters of beam protons, as with this method the scanning efficiency depended only on the scattering angle and the distance of the point of scatter from the emulsion surface. The large number of small-angle scatters originating from interactions with nuclei which gave one- and two-prong stars were therefore used to determine accurately the efficiency for detecting the relatively small number of elastic proton-proton scatters.

With water-loaded emulsion the decrease in thickness caused by processing is greater than for dry emulsion and the apparent increase in scattering angle, θ, caused by this shrinkage is consequently greater, thus improving the efficiency for detecting events by perpendicular line scanning.

1.70 cm² of emulsion were scanned and rescanned using the line-scanning technique and 0.85 cm² of this

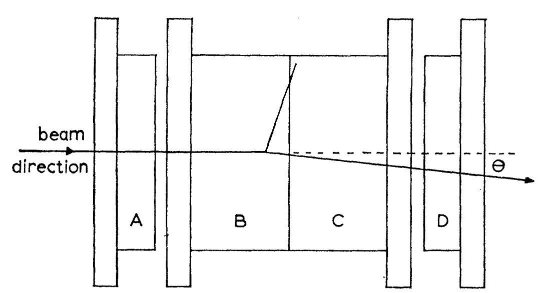

FIG. 1. Schematic diagram of the emulsion positions.

* Present address: Westfield College, London, England.

[1] K_c is the wave number of the incident nucleon and θ_c is the scattering angle in the center-of-mass system.

[2] W. M. Preston, R. Wilson, and J. C. Street, Phys. Rev. **118**, 579 (1960).

[3] V. B. Lyubimov, P. K. Markov, E. N. Tsyganov, Cheng Pu-ing, and M. G. Shafranova, Soviet Phys.—JETP **10**, 651 (1960).

[4] Do In Seb, L. F. Kirillova, P. K. Markov, L. G. Popova, I. N. Silin, E. N. Tsyganov, M. G. Shafranova, B. A. Shakhbazyan, A. A. Yuldashev, Soviet Phys.—JETP **14**, 1243 (1962).

[5] A. I. Zlateva, P. K. Markov, A. T. Peeva, L. G. Khristov, Kh. M. Chernev, Compt. Rend. Acad. Bulgare Sci. **14**, 443 (1961).

[6] S. A. Azimov, Do In Seb, L. F. Kirillova, E. M. Khabibullina, E. N. Tsyganov, M. G. Shafranova, B. A. Shakhbazyan, and A. A. Yuldashev, Soviet Phys.—JETP **15**, 299 (1962).

[7] V. A. Bull, D. A. Garbutt, in Proceedings of the Aix-en-Provence International Conference (Centre d'Etudes Nucleaires de Saclay, Seine et Oise, 1961).

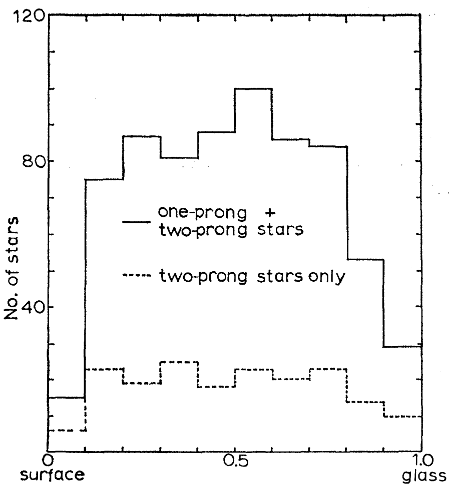

FIG. 2. Depth distribution of one-prong and two-prong stars.

area were area scanned for proton recoils as a further check on efficiency. A total of 650 one-prong and 246 two-prong stars were found, and of these 616 one-prong and 182 two-prong stars lay in the elastic scattering angle range 0–40 mrad. Their depth distribution is shown in Fig. 2. Over the area scanned the average flux density was determined to be $(4.50\pm0.04)\times10^5$ protons/cm^2, the average hydrogen concentration $(6.20\pm0.08)\times10^{22}$ hydrogen atoms/cc, and the product of path length and hydrogen concentration was $(9.58\pm0.14)\times10^{+27}$ cm^{-2}.

SELECTION CRITERIA FOR ELASTIC EVENTS

Four criteria were used to select elastic events from the sample of two prongs stars.

(1) The measured scattering angle, θ_m, should agree with the angle, θ_R, calculated from the range R of the recoil proton, assuming the kinematics of elastic proton-proton scattering.

(2) The incident, scattered and recoil particle momenta should be coplanar.

(3) The measured dip, ϕ_m, of the recoil proton should agree with the dip, ϕ_R, calculated from the range, R, of the recoil proton, assuming the kinematics of elastic proton-proton scattering.

(4) At the point of scatter there should be no nuclear recoil, blob, or electron.

In cases where the recoil proton did not stop in the emulsion, criteria 1 and 3 were replaced by the criterion that the measured dip, ϕ_m, should agree with the angle, ϕ_s, calculated from the measured scattering angle, θ_m, assuming the kinematics of elastic scattering.

MEASUREMENTS

For all two-prong stars the range and dip of the recoil proton, the noncoplanarity angle, γ, and the scattering angle were measured and those which satisfied each of the criteria $1\cdots3$ to within ten standard deviations were measured with higher accuracy.

All ranges were determined to an accuracy of 3% where the error includes range straggling, measurement, and stopping power errors. The stopping power was calculated from the formula given by Barkas *et al.*[8]

$$S=\frac{R_0}{R_d}=\frac{rd-1}{rd_0-1}+\frac{r(d_0-d)R_0}{(rd_0-1)R_w}, \tag{1}$$

where R_0 is the range in standard emulsion of density d_0, R_d is the range in emulsion of density d, R_w is the range in water, and r is the ratio of the volume increment in cc to the weight increment in grams brought about by the addition of water to the emulsion. Within the experimental errors the density of our dry emulsion was equal to d_0 and under this condition Eq. (1) reduces to

$$S=\frac{R_0}{R_w}\left(1-\frac{V_1}{V_2}\right)+\frac{V_1}{V_2}, \tag{2}$$

where V_1 and V_2 are the volumes of the emulsion before and after soaking. The mean range of 40 μ-mesons from π-μ decays agreed with the range calculated using Eq. (2) to better than 1%.

By comparing the directions of the incident and scattered proton tracks with those of six neighboring unscattered beam tracks over a total distance of $\sim$8 mm the scattering angles were measured to an accuracy of ±0.2 mrad.

The dip angles, ϕ_m, of the recoil protons were measured over an optimum cell length chosen to minimize the sum of multiple scattering and setting errors.

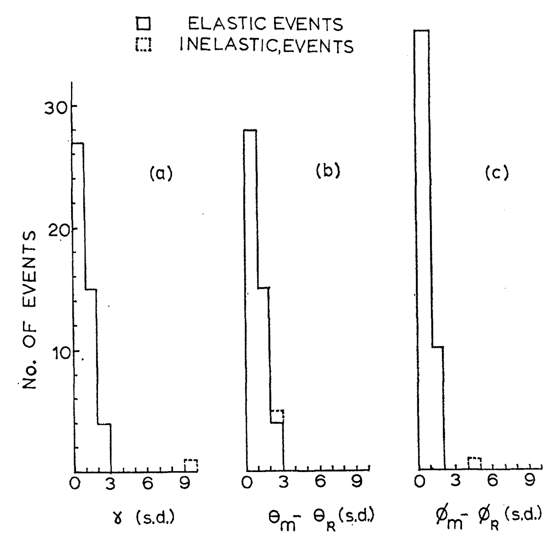

FIG. 3. Distributions of $(\theta_m-\theta_R)$, $(\phi_m-\phi_R)$, and γ for those two-prong stars with scattering angle, θ, in the elastic range and with recoils which stop in the emulsion.

---

[8] W. H. Barkas, P. H. Barret, P. Cuer, H. Heckman, F. M. Smith, and H. K. Ticho, Nuovo Cimento 8, 185 (1958).

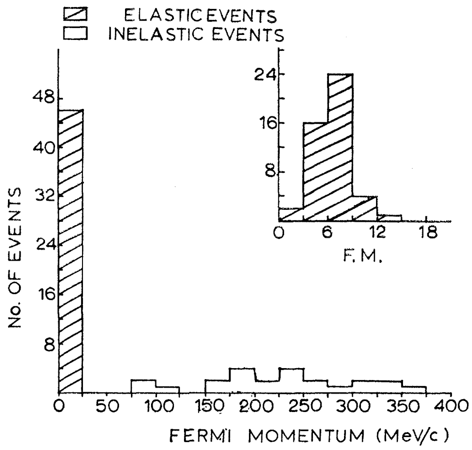

FIG. 4. Distribution of Fermi momentum for two-prong stars with scattering angles in the elastic range and whose recoils stop in the emulsion.

The noncoplanarity error, the sum of the errors in the azimuths of the recoil and scattered protons, was in general less than 1.5°.

APPLICATION OF THE SELECTION CRITERIA AND ESTIMATION OF THE INELASTIC CONTRIBUTION

(i) Figures 3(a), (b), and (c) show the distribution of $(\theta_m-\theta_R)$, $(\phi_m-\phi_R)$, and γ, expressed in standard deviations, for those events, selected as described above, whose recoil protons stopped in the emulsion. Forty-six events satisfied each criteria to within three standard deviations and were defined as elastic.

To estimate the background it was assumed that all the two-prong stars whose recoils stopped in the emulsion and whose scattering angle lay in the elastic range 0–40 mrad were produced by elastic scattering off bound protons. Figure 4 shows the distribution of the Fermi momentum of the bound proton calculated for each event. The measurement errors on the elastic events gave them an apparent Fermi momentum of less than 13 MeV/c. There are no events having Fermi momenta between 13 MeV/c and 75 MeV/c, and the high-momentum tail, above 250 MeV/c, is probably due to events in which neutral particles are produced. Taking the distribution of Fermi momentum to be of the form

$$N(p)dp=Bp^2\exp(-p^2/p_0^2)dp, \tag{3}$$

where $p_0=160$ MeV/c, and normalizing it to the number of inelastic events between 75 and 250 MeV/c the contribution, in the range 0–13 MeV/c, of inelastic events to the elastic peak was less than 0.1 events and considered negligible. This small background is due partly to the decrease in the ratio of inelastic to elastic two-prong stars in water-loaded as compared to dry emulsion[7] and also to the accuracy of measurement which reduces the momentum interval over which the inelastic contribution has to be integrated.

(ii) Figures 5(a) and (b) give the distributions of γ and $(\phi_m-\phi_s)$, expressed in standard deviations, for those accurately measured events whose recoils did not stop in the emuslion and whose scattering angles lay in the elastic range. Twenty-one events satisfied both criteria to within three standard deviations and were defined as elastic.

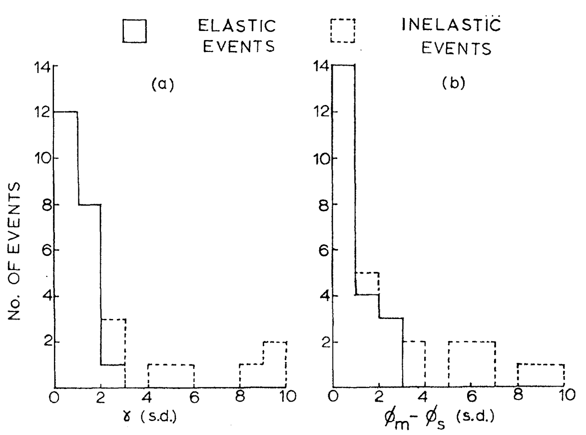

FIG. 5. Distribution of $(\phi_m-\phi_s)$ and γ for those two-prong stars with scattering angles, θ, in the elastic range and whose recoils do not stop in the emulsion.

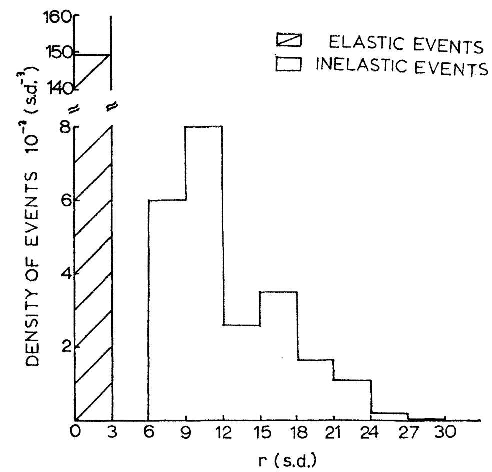

FIG. 6. Density D of inelastic events per unit volume as a function of $r=[(\phi_m-\phi_s)^2+(\theta_R'-\theta_m)^2+\gamma^2]^{\frac{1}{2}}$.

The background in the distribution 5(a) and 5(b) is increased, compared with those in 3(a), (b), and (c). For these events, criterion (1) cannot be applied, and because of the high momentum transfers, the departure from elasticity caused by the presence of Fermi momentum is less marked. However, from the available range in the emulsion, it is possible to obtain a lower limit θ_R' to θ_R so that if $\theta_m<\theta_R'$, then $\theta_R'-\theta_m$ may be used as a lower limit in the application of criterion (1). As Fermi momentum is an unknown parameter for these events, in order to estimate the background it is necessary to consider all the criteria simultaneously. If a three-dimensional figure were constructed having axes $(\theta_R'-\theta_m)$, $(\phi_m-\phi_s)$, and γ, a clustering of elastic events near the origin is expected with the inelastic events spread throughout the volume. The elastic events lie inside a cube of side three standard deviations and the background is determined from the density of inelastic events in this cube. Figure 6 shows the density (D) of inelastic events per unit volume plotted as a function of the distance $r=[(\phi_m-\phi_s)^2+(\theta_R'-\theta_m)^2+\gamma^2]^{1/2}$. The density D decreases as $r\to 0$ for $r<10$ but even if the background is overestimated by taking the maximum density as the value inside the cube the inelastic contribution is found to be only 0.2 events.

SCANNING EFFICIENCY

Sixty-three elastic events were found using the line-scanning method and an additional 4 events, all in the top 0.1 of the emulsion thickness, were found using the area scan. The depth distribution of these events is shown in Fig. 7. Only the 55 events in the region 0.1–0.8 of the emulsion thickness were subsequently used in the calculation of the total and differential cross sections because of the scanning losses which occur in the top 0.1 and bottom 0.2 of the emulsion (see Figs. 2 and 7). The scanning efficiency was determined from those one- and two-prong stars in the fiducial volume whose scattering angles lay in the elastic range, by dividing them into four angular intervals according to the scattering angle of the fast track. From the number of events found in both scans and those events found in one scan but otherwise missed, an efficiency for each angular interval was calculated. The efficiency was 82% for scattering angles between 3 and 6.5 mrad and was approximately constant at 92% for larger angles.

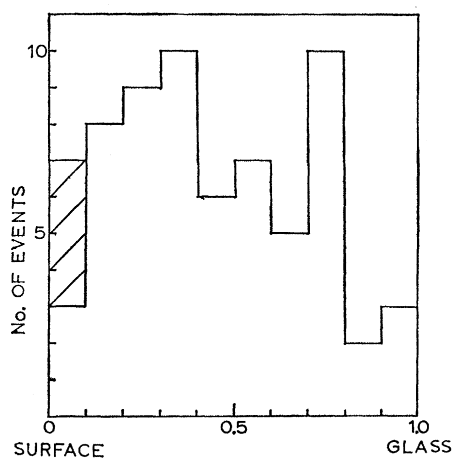

FIG. 7. Depth distribution of elastic p-p scattering events.

TOTAL CROSS SECTION

For angles less than 3 mrad the scanning efficiency is low and the observed cross sections have to be corrected for the contribution from Coulomb scattering. As the determination of such a contribution requires assumptions concerning the nature of the interference between nuclear and Coulomb scattering, a cutoff in scattering angle was applied at 3 mrad at which value the Coulomb scattering is ~4% of the nuclear. The remaining 53 events correspond to an observed cross section of 7.9 mb.

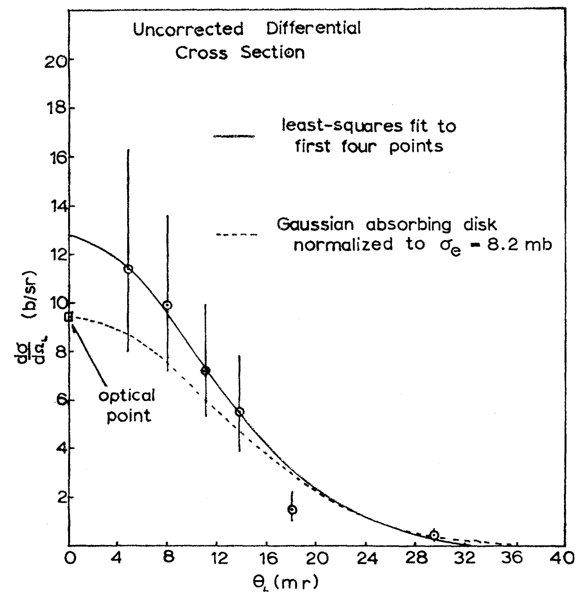

FIG. 8. Differential cross section uncorrected for scanning losses. The solid curve is fitted to the experimental points at the four smallest angles. The dashed curve passes through the optical point and is normalized to the observed total cross section.

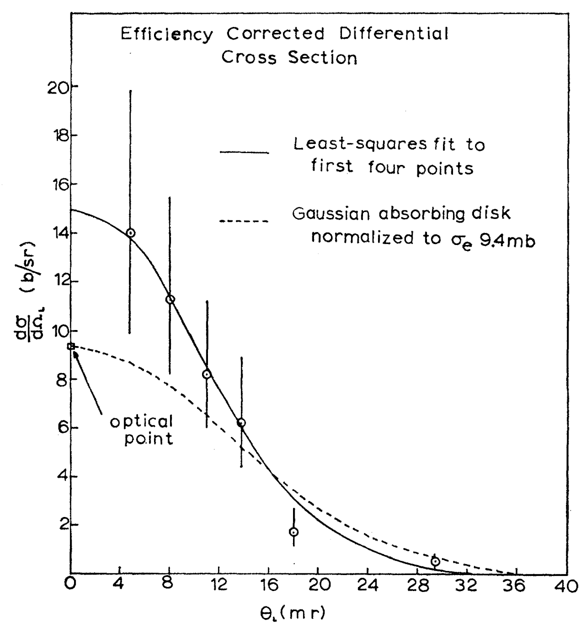

FIG. 9. Differential cross section corrected for scanning losses. The solid curve is fitted to the experimental points at the four smallest angles. The dashed curve passes through the optical point and is normalized to the corrected total cross section.

The contribution to the elastic cross section from events whose scattering angles were less than the cutoff value was calculated, using the least-square fit to the data as described below, as 0.36 mb. To allow for the inelastic contribution, 0.05 mb was subtracted. The resultant total cross section uncorrected for scanning efficiency was

$$\sigma_e = 8.2 \pm 1.1 \text{ mb}.$$

If scanning losses are taken into account and corrections, as described above, are again applied, the corrected total cross section becomes 9.4 ± 1.3 mb. This is in agreement with the value 10.0 ± 1.8 mb. reported by Diddens *et al.*[9] at 18.6 GeV/c.

DIFFERENTIAL CROSS SECTION

Figures 8 and 9 give the uncorrected and efficiency corrected differential cross sections for scattering angles greater than 3 mrad. To obtain extrapolated zero-angle cross sections unbiased by the measurements at large angles, Gaussian functions have been used to fit the data at angles less than 15 mrad. The extrapolations gave

$$(d\sigma/d\Omega_L)(0°) = 12.8_{-2.3}^{+2.8} \text{ b/sr} \quad \text{(uncorrected)},$$
$$= 15.0_{-2.6}^{+3.2} \text{ b/sr} \quad \text{(corrected)}.$$

These cross sections are to be compared with the value of 9.4 ± 1.3 b/sr predicted by the optical theorem using a total cross section of $\sigma_T = 39.7 \pm 1.5$ mb.[10]

DISCUSSION

If the scattering amplitude for elastic proton-proton scattering is purely imaginary and spin-independent, the optical theorem predicts that the zero-angle differential cross section is given by

$$(d\sigma/d\Omega_L)(\text{opt}) = [K_L\sigma_T/4\pi]^2. \quad (4)$$

Figures 8 and 9 compare the experimental values of the differential cross sections with those given by the equation for a Gaussian absorbing disk of radius R:

$$\frac{d\sigma}{d\Omega_L} = \left[\frac{K_L\sigma_T}{4\pi}\right]^2 \exp\left[-\frac{K_L^2R^2\theta_L^2}{2}\right]. \quad (5)$$

The curve given by this equation is normalized to the observed total elastic cross section, uncorrected (Fig. 8) and corrected (Fig. 9) for scanning losses. At small scattering angles the experimental points all lie above the theoretical curves and the least-squares fits give for the ratio of the zero-angle cross section to the optical value

$$\frac{d\sigma}{d\Omega_L}(0^0) \Big/ \frac{d\sigma}{d\Omega_L}(\text{opt}) = 1.36_{-0.24}^{+0.30} \quad \text{(uncorrected)},$$
$$= 1.60_{-0.28}^{+0.34} \quad \text{(corrected)}.$$

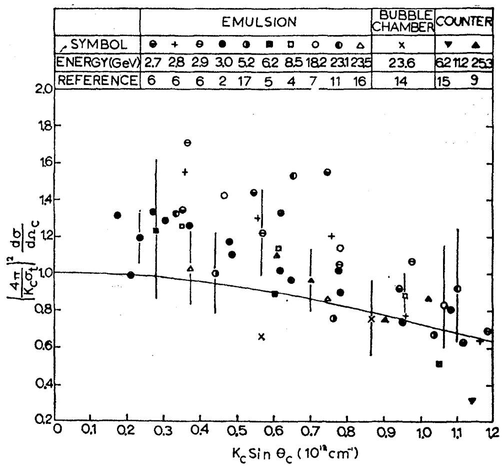

FIG. 10. Differential cross sections in the range 2.5–25 GeV references 2, 4–7. The solid curve is that for a purely absorbing Gaussian disk normalized to a total elastic cross section of 10 mb.

There have been a number of experiments above 2.5 GeV in which the authors[2–6,9,11] have also reported small angle differential cross sections in excess of that given by the optical theorem. A critical discussion of these experiments is given in the review papers of Marquit.[12,13] To compare such proton-proton scattering experiments at different energies it is convenient to express the measured differential cross sections, in the form $[4\pi/K_c\sigma_T]^2(d\sigma/d\Omega_c)$ as a function of $K_c \sin\theta_c$. In this form the differential cross sections should coincide at all energies for $K_c \sin\theta_c < 1.2 \times 10^{13}$ cm^{-1} as in this region the differential cross section is insensitive to the parameters of the optical model chosen to describe the scattering. Figure 10 shows the results of all experiments[9,11,14–17] in the range 2.5–25 GeV which quote cross sections for values of $K_c \sin\theta_c < 1.2 \times 10^{13}$ cm^{-1}.[18] They are compared with the cross section given by Eq. (5) with R chosen to give a total elastic cross section of 10 mb.[9] As the cross sections are compared with a purely absorbing model the Coulomb scattering cross section has been directly subtracted from the measured values. This correction is negligible, however, for values of $K_c \sin\theta_c > 0.4 \times 10^{13}$ cm^{-1}. To avoid confusion, only a few representative errors have been given, but approximately 50% of the points lie more than one standard deviation

[9] A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **9**, 32 (1962).

[10] A. Ashmore, G. Cocconi, A. N. Diddens, and A. M. Wetherell, Phys. Rev. Letters **5**, 576 (1960).

[11] E. Marquit (private communication).

[12] E. Marquit, Report No. 255/VI, Institute of Nuclear Research, Polish Academy of Sciences, Warsaw, (1961).

[13] E. Marquit, Phys. Letters **1**, 41 (1962).

[14] D. R. O. Morrison, International Conference on the Theoretical Aspects of High Energy Phenomena. CERN (1961).

[15] B. Cork, W. A. Wenzel, and C. W. Causey, Jr., Phys. Rev. **107**, 859 (1957).

[16] G. Czapek, G. Kellner, and F. Otter (private communication).

[17] E. N. Tsyganov, Soviet Phys.—JETP **15**, 1009 (1962).

[18] See also B. Bekker, L. Kirillova, A. Nomofilov, V. Nikitin, V. Pantuev, V. Sviridov, L. Strunov, M. Khachaturian, and M. Shafranova, in *Proceedings of the 1962 Annual International Conference on High Energy Physics at CERN* (CERN, Geneva, 1962). J. Fujii, G. B. Chadwick, G. B. Collins, P. J. Duke, N. C. Hien, M. A. R. Kemp, and F. Turkot. Phys. Rev. **128**, 1823 (1962).

above the theoretical curve through the optical point. Figure 11 gives the distribution of experimental points as a function of the difference (d), expressed in standard deviations between the experimental cross sections and those given by Eq. (5), with the shaded portion representing the distribution of those points for which the Coulomb correction is negligible. The marked asymmetry of both distributions indicates that a purely absorbing model does not adequately describe the experimental data in this region. A least-squares fit to the experimental points of Fig. 10, using an equation of the form

$$\left[\frac{4\pi}{K_c\sigma_T}\right]^2\frac{d\sigma}{d\Omega_c}=A\exp(-B\theta_c^2), \tag{6}$$

gave

$$A=1.27\pm0.11.$$

This result would follow if there were a systematic underestimation of the total p-p cross section of 13% throughout the entire energy range whereas the quoted errors are less than 3%. It appears, therefore, that the available data on small-angle proton-proton scattering above 2.5 GeV require for their explanation a real part of the potential or, as pointed out by Veksler[19] and discussed in detail by Azimov *et al.*,[6] a spin dependence of the p-p cross section.

Measurements of the total antiproton-proton cross section show that it is slowly decreasing with energy above 10 GeV and even at 20 GeV it is still 6 mb above the total proton-proton cross section which is constant at ~40 mb at these energies. It appears therefore that the Pomeranchuk limit has not been reached and dispersion relations[20] show that only in this limit does the ratio of the real to the imaginary part of the forward scattering amplitude approach zero. It may be, therefore, that at 20 GeV the real part of the scattering amplitude is still significant. The existence of the real part and the high-energy behavior of p-p and $\bar{p}$-p total cross sections may be explained by a Regge-pole model which includes ω and P' trajectories in addition to the vacuum pole trajectory P (for example, see Frautschi *et al.*[21]). Using this model Hadjioannou *et al.*[22,23] have calculated the ratio A of the forward p-p cross section to the optical theorem limit as a function of energy. In Table I this ratio is compared with the experimentally determined values of A taken from references 2, 4, 6, and 9 in which extrapolations to zero angle have been made. Although theory and experiment are not in contradiction, the measured values of A above 7 GeV, which is considered by Hadjioannou *et al.* to be the lower limit of validity of the theory, are all greater than the predicted values and do not appear to be converging to the optical limit.

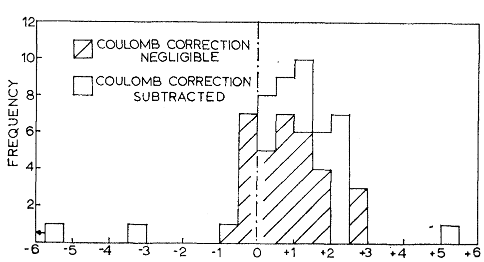

FIG. 11. Distribution of the experimental points of Fig. 10 as a function of the difference between the experimental cross sections and those given by Eq. (5).

TABLE I. Comparison of the experimentally determined and theoretically predicted values of A.

Momentum (GeV/c)		A Experimental	Theoretical
3.7		1.46 ± 0.12[a]	1.44
3.8		1.22 ± 0.12[b]	1.43
9.1		1.36 ± 0.11[c]	1.18
12.1–26.2		1.2 ± 0.2[d]	1.14–1.07
19.2	(uncorrected)	$1.36_{-0.24}^{+0.30}$	1.08
	(corrected)	$1.60_{-0.28}^{+0.34}$	

[a] See reference 6.
[b] See reference 2.
[c] See reference 4.
[d] See reference 9.

CONCLUSION

The present experiment obtains a total cross section for elastic proton-proton scattering of 9.4 ± 1.3 mb and an extrapolated zero-angle differential cross section in excess of that predicted by the optical theorem. This excess does not disagree with a three-trajectory Regge-pole model.

ACKNOWLEDGMENTS

We wish to thank the members of the CERN emulsion group and the P.S. staff for their help and the facilities put at our disposal during the exposure of the stack, Professor R. D. Hill for helpful discussions and for designing the clamp used in the investigation and Dr. R. J. N. Phillips for valuable discussions on the Regge-pole theory. We are indebted to Mrs. F. Beniston, Miss C. Christodoulou, Mrs. M. Culhane, and Miss E. Osborne for their efficient scanning of the stack. Acknowledgment is also made of D.S.I.R. studentships to V.A.B. and D.A.G. Finally, we would like to thank Professor E. H. S. Burhop for his constant advice and encouragement.

[19] V. I. Veksler, Nucleon-nucleon and Pion-nucleon Interaction Report in *Proceedings of the 1959 Annual International Conference on High Energy Physics at Kiev.* (Kiev, U.S.S.R., 1959).
[20] H. Lehmann, Nucl. Phys. **29**, 300 (1962).
[21] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).
[22] F. Hadjioannou, R. J. N. Phillips, and W. Rarita, Phys. Rev. Letters **9**, 183 (1962).
[23] R. J. N. Phillips (private communication).